J. Fluid Mech. (1976), vol. 76, part 3, pp. 457–480
Printed in Great Britain

On the cause and characteristic scales of meandering and braiding in rivers

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(Received 11 April 1975 and in revised form 28 January 1976)

A stability analysis of meandering and braiding perturbations in a model alluvial river is described. A perturbation technique, involving a small parameter representing the ratio of sediment transport to water transport, is used to obtain the following results.

Under appropriate conditions, the existence of sediment transport and friction are necessary conditions for the occurrence of instability in the flow and on the bed; thus instability is not inherent in the flow alone. An Anderson-type scale relation for longitudinal instability is obtained for meandering. A relation estimating the number of braids and differentiating between meandering and braided regimes is derived. These relations are independent of sediment transport.

1. Introduction

River channels possess three characteristic fluvial morphologies: straight, meandering and braided. The latter two states are illustrated in figure 1 (plate 1). Deviation from the straight configuration appears to be associated with a fluvial instability (Callander 1969). The cause of the instability has been the subject of much speculation. Such investigators as Leopold & Wolman (1957), Gorycki (1973*a*, *b*) and Karcz (1971) have suggested that meandering is inherent in the flow, the role of sediment transport being passive and collateral. In fact meandering has been observed not only in alluvial rivers, but also in supra-glacial melt-water streams, in the Gulf Stream and Kuroshio, and in the small laboratory streams (a few millimetres in width) of Gorycki; in each of these cases, sediment transport is absent. Others, notably Schumm (1963), have emphasized the role of sediment in the meandering mechanism. Similarly, the studies of Hayashi (1970) and Werner (1951) have given resistance a minor role, whereas Hansen (1967) and Gorycki have asserted the opposite.

Many theories have been developed to predict a commonly measured parameter of stream morphology: the longitudinal meander or braid wavelength. In the case of meandering, the following three formulae are typical. Werner considered gravity waves and obtained $\lambda/B \sim 2F$, where λ is the wavelength, B is the stream breadth and F is the flow Froude number. Hansen used a stability model to obtain $\lambda/d_0 = 7F^2S^{-1}$, where d_0 is the flow depth and S is the longitudinal channel slope. Anderson (1967) analysed transverse oscillations through a pendulum analogy and obtained

$$\lambda/(Bd_0)^{\frac{1}{2}} = \text{constant} \times F^{\frac{1}{2}},$$

where the constant is evaluated from flume data as being about 72. These formulae predict very different wavelengths.

In recent years stability models have succeeded in explaining a number of aspects of meandering and braiding. In addition to Hansen, Callander and Hayashi, Adachi (1967), Sukegawa (1970) and Engelund & Skovgaard (1973) have made notable contributions.

The present paper considers a two-dimensional stability model due to Hansen and Callander. Expansion techniques are used to obtain an analytical description, the essential results being a differentiation between meandering and braided regimes, derivation of relations for the meander wavelength and number of braids, and a restricted establishment of sediment transport as a necessary condition for the occurrence of fluvial instability.

2. The theoretical model river

Treatment of actual rivers presents insurmountable difficulties, so in order to expedite the analysis an idealized model river which retains those features essential for the occurrence of fluvial instability is proposed.

The following statements are observed to be true in nature and have been demonstrated in laboratory flumes: (i) meandering and braided rivers generally have large width–depth ratios; (ii) the sinuous pattern of banks and bars is due to the emergence of a submerged alternating bar pattern on the bed of an otherwise roughly straight river during low flow; (iii) these alternating bars occur even in channels with non-erodible banks. Justification of these statements for natural rivers is provided by Kinoshita (1957) and Fahnestock & Maddock (1964).

The implication here is that a theory of submerged alternating bar formation in straight rivers is also a theory of the origins of meandering and braiding. Figure 2 (plate 2) shows two stages in the development of a meander pattern in an initially straight channel at the St Anthony Falls Hydraulic Laboratory. First an alternating bar pattern develops while the channel is still straight, but in time lateral growth of the bars causes bank erosion and channel meandering, preserving the wavelength of the alternating bars.

The existence of initial channel curvature and the secondary flow associated with it has been observed to expedite the meandering process (Friedkin 1945). However, initial curvature is not necessary for the formation of either meandering or braiding, and both form readily in its absence, as is illustrated in figure 2 (Schumm & Khan 1972; Anderson, Parker & Wood 1975).

Thus an investigation into the cause of fluvial instability can be carried out by theoretically examining submerged bedforms in a model shallow channel with width B and with straight, non-erodible, parallel, vertical banks. Except for the bars, the channel cross-section is assumed rectangular. The channel is of constant slope S and carries a constant discharge Q. The bed consists of uniform erodible (or non-erodible) material, and for simplicity it is also postulated that the stream carries no suspended load.

This theoretical model river is realized in solid-walled tilting flumes of the recirculating or sediment-feed type.

3. The momentum and mass balance equations

The shallowness of the assumed model suggests a two-dimensional (i.e. excluding the vertical direction) approach to momentum and mass balance in which velocities are assumed to be quantities averaged over the depth. Postulating hydrostatic pressure, the general balance equations are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial h}{\partial x}-\frac{\tau_x}{\rho d},$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-g\frac{\partial h}{\partial y}-\frac{\tau_y}{\rho d},$$

$$\frac{\partial}{\partial x}(ud)+\frac{\partial}{\partial y}(vd)+\frac{\partial d}{\partial t}=0,$$

$$\frac{\partial (h-d)}{\partial t}+\frac{1}{1-\lambda_p}\left(\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}\right)=0,$$

where d is the channel depth, h is the water surface's height above some reference level, (u, v) is the stream velocity vector, (q_x, q_y) is the vector of volumetric sediment transport (bed load), (τ_x, τ_y) is the bed stress vector, ρ is the density of water and λ_p is the bed porosity.

This two-dimensional approach ignores the effect of dynamic pressure and only partially accounts for helicity effects induced by the vertical velocity component. The former is justified, whereas the latter may not be. A more complete, three-dimensional approach is outlined in Engelund & Skovgaard (1973).

The momentum equations can be written in shorthand tensor notation in the form

$$\tau_i = \rho g d \Sigma_i, \quad i = 1, 2,$$

where

$$\Sigma_i = -\left(\frac{\partial h}{\partial x_i}+\frac{\partial u_i}{g\,\partial t}+\frac{u_j\,\partial u_i}{g\,\partial x_j}\right). \tag{1}$$

Note that Σ_i is a generalized slope vector.

4. Constitutive relations

Perturbations about a steady flow are to be considered; however a knowledge of the steady flow is required first. The balance equations admit the following unperturbed solution: $v = q_y = \tau_y = 0$, $u_x = U$, $q_x = q_0$, $\tau_x = \tau_0$, $h = h_0 - Sx$ and $d = d_0$. Here the following constraints must be satisfied: if $\mathscr{q} = Q/B$ and $C_0 = \tau_0/\rho U^2$,

$$\mathscr{q} = U d_0, \quad C_0 U^2 = g d_0 S. \tag{2), (3}$$

Note that C_0 is a dimensionless friction factor and $\mathscr{q}$ is the water discharge per unit width.

In order to specify the unperturbed flow, constitutive relations for C_0 and q_0 must be known. These relations have been a major source of controversy in the field of sediment hydraulics, so herein a theory general enough to accommodate any specific load or resistance relation is presented.

First it is assumed that enough constraints exist so that solution sets (unique or otherwise) can indeed be specified. (Concurrence on this point is far from universal; see Maddock 1970.) In a given model river, the relevant parameters are B, $\mathscr{q}$, d_0, U, q_0, S, C_0, g, the characteristic sediment diameter d_s, the fluid kinematic viscosity ν, the fluid and particle densities ρ and ρ_s, and the dimensionless sediment shape, distribution and porosity factors p_i, $i = 1, 2, 3, \ldots$. The following set of dimensionless parameters can be specified from these:

$$\psi = \left(\frac{\rho_s}{\rho} - 1\right), \quad S, \quad F_s = \frac{U}{(\psi g d_s)^{\frac{1}{2}}}, \quad R_p = \frac{(\psi g d_s)^{\frac{1}{2}} d_s}{\nu},$$

$$q_f = \frac{\mathscr{q}}{(\psi g d_s)^{\frac{1}{2}} d_s}, \quad q^* = \frac{q_0}{(\psi g d_s)^{\frac{1}{2}} d_s}, \quad C_0, \quad R = \frac{d_0}{d_s},$$

$$d_0/B, \quad p_i, \quad i = 1, 2, 3, \ldots .$$

The parameter d_0/B is assumed small and is thus ignored. The two constraints (2) and (3) can be used to eliminate two more parameters, leaving, for example, F_s, S, R_p, ψ, C_0, q^* and p_i as an independent set. From the remaining parameters, the most general possible (but not necessarily single-valued) constitutive relations for resistance and sediment transport can be formed:

$$C_0 = \phi_c(F_s, S; R_p, \psi, p_i), \tag{4}$$

$$q^* = \phi_q(F_s, S; R_p, \psi, p_i). \tag{5}$$

Relations equivalent to these have been determined empirically by Peterson (1975). Also, any specific pair of resistance and load equations can be cast in the above form. *The utility of this formulation in the present work is that the* stability model presented below is rendered independent of specific resistance and load relations.

With (4) and (5) two more parameters are eliminated, leaving five free parameters. A knowledge of water temperature and sediment type specifies ψ, R_p and p_i. The final pair of constraints needed to specify the unperturbed flow varies depending on external conditions. In a sediment-feed flume, q_f and q^* are known. In nature the constraints are complex, but for graded rivers (rivers in long-term equilibrium) q_f and S are easily measured.

Thus, in principle, the unperturbed flow can be specified. In practice a numerical procedure, dependent on a specific evaluation of (4) and (5), is usually required. Henceforth it is assumed that the constitutive relations are known.

The constitutive relations are now generalized for slightly varying flow. For clarity, tensor notation is used in this section alone; for example, u_i, $i = 1$ or 2, indicates a component of the velocity vector $\mathbf{u}$ and u indicates the magnitude of

$\mathbf{u}$, i.e. $u^2 = u_i u_i$, rather than the x component. Equations (4) and (5) are replaced by general relationships:

$$\tau_i/\rho u^2 = L_i^1(u_j/(\psi g d_s)^{\frac{1}{2}}, \Sigma; x_j, t), \tag{6}$$

$$q_i/(\psi g d_s)^{\frac{1}{2}} d_s = L_i^2(u_j/(\psi g d_s)^{\frac{1}{2}}, \Sigma; x_j, t), \tag{7}$$

where R_p, ψ and p_i are suppressed since they are constants for a given river reach, $\Sigma^2 = \Sigma_i \Sigma_i$, and L_i^1 and L_i' are unevaluated operators. Assuming isotropy and spatial homogeneity, these relations reduce to

$$\tau_i = \rho C u u_i, \quad q_i = (\psi g d_s)^{\frac{1}{2}} d_s q^* u^{-1} u_i, \tag{8), (9}$$

where C and q^* can be obtained directly from (4) and (5), which have been previously evaluated for non-varying flow, as

$$C = \phi_c(u/(\psi g d_s)^{\frac{1}{2}}, \Sigma), \quad q^* = \phi_q(u/(\psi g d_s)^{\frac{1}{2}}, \Sigma).$$

The two-dimensional generalization provided by these relations is essentially the same as that used implicitly by Callander and Hansen. Henceforth tensor notation is replaced by the previous notation.

5. The perturbed equations of motion

The balance equations are considered for slight perturbations about steady uniform flow, i.e.

$$(u, v) = (U, 0) + (u', v'),$$
$$(q_x, q_y) = (q_0, 0) + (q_x', q_y'),$$
$$(\tau_x, \tau_y) = (\tau_0, 0) + (\tau_x', \tau_y'),$$
$$h = h_0 - Sx + h', \quad d = d_0 + d',$$

where, for example, $(u'^2 + v'^2)^{\frac{1}{2}}/U \ll 1$. This yields, with appropriate linearization,

$$\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} = -g\frac{\partial h'}{\partial x} + \frac{\tau_0}{\rho d_0^2} d' - \frac{\tau_x'}{\rho d_0},$$

$$\frac{\partial v'}{\partial t} + U\frac{\partial v'}{\partial x} = -g\frac{\partial h'}{\partial y} - \frac{\tau_y'}{\rho d_0},$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{1}{d_0}\left(\frac{\partial d'}{\partial t} + U\frac{\partial d'}{\partial x}\right) = 0,$$

$$\frac{\partial (h'-d')}{\partial t} + \frac{1}{1-\lambda_p}\left(\frac{\partial q_x'}{\partial x} + \frac{\partial q_y'}{\partial y}\right) = 0.$$

The constitutive relations (8) and (9) are likewise linearized by means of double Taylor series expansions about the unperturbed slope and velocity; for example

$$\tau_x' = 2\rho C U\left(1 + \frac{1}{2}\frac{U}{C}\frac{\partial C}{\partial U}\right) u' + \rho\frac{\partial C}{\partial S} U^2 \Sigma',$$

where Σ' is the perturbed part of Σ, and where C, $\partial C/\partial U$ and $\partial C/\partial S$ are evaluated in the unperturbed state, i.e. $u^2 + v^2 = U^2$, $\Sigma = S$ and $C = C_0$. A reduction of these relations provides the determinate perturbed balance equations, which are presented in dimensionless form according to the following transformation, designed to avoid bulky starred equations: $x \to x^*$, $y \to y^*$, $t \to t^*$, then $x^* = d_0 x$, $y^* = d_0 y$, $t^* = (d_0/U)t$, $u' = Uu$, $v' = Uv$, $h' = d_0 h$ and $d' = d_0 d$. The resulting equations are

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x} = -F^{-2}\frac{\partial h}{\partial x}-C_0(M_1 u - M_2 d), \tag{10a}$$

$$\frac{\partial v}{\partial t}+\frac{\partial v}{\partial x} = -F^{-2}\frac{\partial h}{\partial y}-C_0 v, \tag{10b}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial d}{\partial t}+\frac{\partial d}{\partial x} = 0, \tag{10c}$$

$$\frac{\partial}{\partial t}(h-d)+\beta\left(N_1\frac{\partial u}{\partial x}-N_2\frac{\partial d}{\partial x}+\frac{\partial v}{\partial y}\right) = 0. \tag{10d}$$

The constants appearing in these equations are the undisturbed Froude number $F = U/(gd_0)^{\frac{1}{2}}$, a dimensionless measure of sediment transport $\beta = q_0/(1-\lambda_p)\,U d_0$, and

$$M_1 = 2\left(1-\frac{S}{C_0}\frac{\partial C_0}{\partial S}\right)^{-1}\left(1+\frac{1}{2}\frac{U}{C_0}\frac{\partial C_0}{\partial U}\right),$$

$$M_2 = \left(1-\frac{S}{C_0}\frac{\partial C_0}{\partial S}\right)^{-1},$$

$$N_1 = \frac{U}{q_0}\frac{\partial q_0}{\partial U}+\frac{S}{q_0}\frac{\partial q_0}{\partial S}M_1,$$

$$N_2 = \frac{S}{q_0}\frac{\partial q_0}{\partial S}M_2.$$

Both their form and specific evaluation indicate that the constants M_1, M_2, N_1 and N_2 are of order unity.

6. The characteristic polynomial

To conduct a stability analysis, the following generalized dimensionless sinusoidal perturbations are introduced into the balance equations:

$$\left.\begin{aligned} u &= \hat{u}(y)\exp i(kx-\phi t), \quad v = \hat{v}(y)\exp i(kx-\phi t),\\ h &= \hat{h}(y)\exp i(kx-\phi t), \quad d = \hat{d}(y)\exp i(kx-\phi t).\end{aligned}\right\} \tag{11}$$

Here k is the dimensionless fluvial instability wavenumber, related to the dimensional wavelength λ by the relation $\lambda/d_0 = 2\pi/k$, and ϕ is the complex celerity, its real part ϕ_r being related to the disturbance wave speed c by $c = \phi_r/k$ and its imaginary part ϕ_i being the temporal exponential growth rate. Note that instability requires that $\phi_i > 0$.

These perturbations are introduced into the balance equations and are reduced with the aid of the boundary condition of impermeable banks; i.e., if

$B^* = B/d_0$, $v = 0$ at $y = \pm\frac{1}{2}B^*$. As a result it is found that $\hat{v}$ can take only the forms

$$\hat{v} \sim \begin{cases} \cos(\frac{1}{2}mk_B y), & m = 1, 3, 5, \ldots, \\ \sin(\frac{1}{2}mk_B y), & m = 2, 4, 6, \ldots, \end{cases} \tag{12}$$

where $k_B = 2\pi/B^*$ is the dimensionless width wavenumber.

The bed height fluctuation is given by $\eta = h-d$; thus $\hat{\eta} = \hat{h}-\hat{d}$. Reduction shows that $\hat{\eta} \sim \partial\hat{v}/\partial y$. Thus the bed patterns corresponding to various values of m can be deduced from (11). For $m = 1$ the bed pattern consists of a single braid of submerged alternating bars characteristic of the early stages of meandering, as may be seen in figure 3. Increasing values of m imply an increased tendency towards incipient braiding, with m equalling the number of braids, as can be seen from the appearance of middle bars in figure 3 for values of m greater than one.

The complex celerity ϕ must satisfy the dispersion equation implied by (10) and (11):

$$\begin{aligned}\phi^4 &+ [-3k+iC_0(M_1+1)]\,\phi^3 + [-K^2(1+\beta) - M_1C_0^2 + k^2(3-F^{-2}\{1+N_1\beta\}) \\ &- iC_0(2M_1+M_2+2)\,k]\,\phi^2 + [(K^2\{1+\beta(2+N_2)\} + \{M_1+M_2\}\,C_0^2)\,k \\ &- (1-F^{-2}\{1+\beta(2N_1+N_2)\})\,k^3 + iC_0(-K^2M_1\{1+\beta\} + k^2\{(M_1+M_2+1) \\ &- F^{-2}(1+N_1\beta)\})]\,\phi + F^{-2}\beta[-F^2K^2(1+N_2)\,k^2 - k^4(N_1+N_2) \\ &\qquad + iC_0(F^2K^2k\{M_1+M_2+N_2M_1-N_1M_2\} + \{N_1+N_2\}\,k^3)] = 0,\end{aligned}$$

in which $K = \frac{1}{2}mF^{-1}k_B$. To test for instability it is necessary to obtain the four roots of this polynomial equation for the complex celerity.

The form of the dispersion equation is $f(\phi, k, K, F, C_0, \beta, M_1, M_2, N_1, N_2) = 0$. The number of parameters is large, and a general solution for ϕ is prohibitively tedious. Here asymptotic expansions in the small parameter β are considered, i.e. $\phi = \phi_0+\beta\phi_1+\beta^2\phi_2+\ldots$. In fact β, which is essentially the ratio of bed sediment transport to fluid transport, is almost universally small. Typical values for various laboratory and field rivers are given in table 1. The equation for the lowest-order term in the expansions for three roots of the dispersion equation (the subscript has been omitted) is

$$\begin{aligned}\phi^3 &+ [-3k+iC_0(M_1+1)]\,\phi^2 + [-K^2 + k^2(3-F^{-2}) - M_1C_0^2 \\ &- iC_0(2M_1+M_2+2)\,k]\,\phi + [(K^2+\{M_1+M_2\}\,C_0^2)\,k - (1-F^{-2})\,k^3 \\ &\qquad + iC_0(-K^2M_1 + k^2\{(M_1+M_2+1) - F^{-2}\})] = 0.\end{aligned} \tag{13}$$

The equation for the lowest-order term in the expansion for the fourth root (subscript omitted) is

$$\begin{aligned}[(K^2 &+ \{M_1+M_2\}\,C_0^2)\,k - (1-F^{-2})\,k^3 + iC_0(-K^2M_1 + k^2\{(M_1+M_2+1) \\ &- F^{-2}\})]\,\phi + F^{-2}\beta[-F^2K^2k^2(1+N_2) - k^4(N_1+N_2) \\ &\qquad + iC_0(F^2K^2k\,\{M_1+M_2+N_2M_1-N_1M_2\} + \{N_1+N_2\}\,k^3)] = 0.\end{aligned} \tag{14}$$

The above decomposition of the dispersion equation has an important physical meaning. The three roots obtainable from (13) will be independent of the

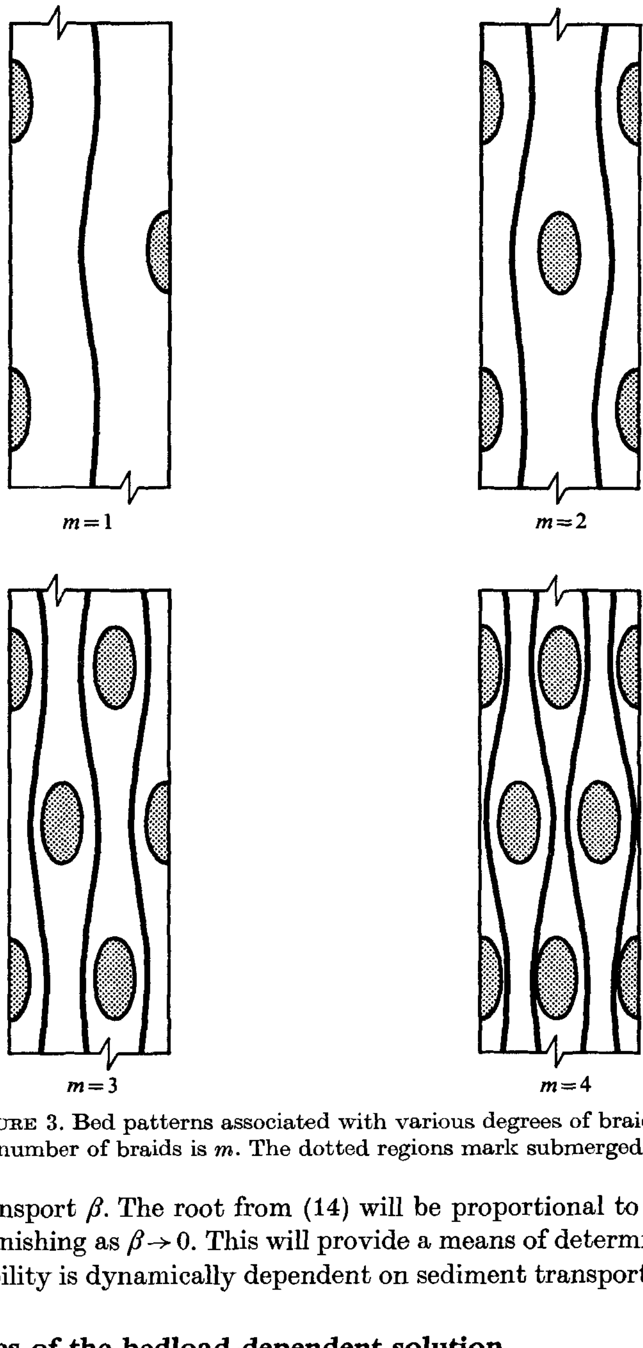

FIGURE 3. Bed patterns associated with various degrees of braiding. The number of braids is m. The dotted regions mark submerged bars.

sediment transport β. The root from (14) will be proportional to the sediment transport, vanishing as $\beta \rightarrow 0$. This will provide a means of determining whether fluvial instability is dynamically dependent on sediment transport.

7. Properties of the bedload-dependent solution

The imaginary part of the solution to (14) is

$$\phi_i = \beta C_0 \frac{(A_{11}K^4 + A_{12}C_0^2K^2)\,k^2 + (A_{21}K^2 + A_{22}C_0^2)\,k^4 + A_3k^6}{A_4C_0^2K^4 + (A_{51}K^4 + A_{52}C_0^2K^2 + A_{53}C_0^4)\,k^2 + (A_{61}K^2 + A_{62}C_0^2)\,k^4 + A_7k^6}, \tag{15}$$

Investigator(s)	River	State	Typical value of β
Hubbell & Matejka (1959)	Middle Loup River	Braided	$2\cdot5\times10^{-4}$
Jones, Hawley & Crippen (1972)	Clear Creek	Meandering	1×10^{-6}
Mapes (1969)	Walla Walla River	Meandering	$2\cdot5\times10^{-4}$
Nordin & Beverage (1965)	Rio Grande	Braided	$2\cdot5\times10^{-4}$
Colby & Hembree (1955)	Niobrara River	Meandering	5×10^{-4}
Fahnestock (1963)	White River	Braided	1×10^{-3}
Sharma (1974)	Laboratory	Meandering	1×10^{-3}
Quraishy (1973)	Laboratory	Meandering	1×10^{-3}
Schumm & Kahn (1972)	Laboratory	Braided	$2\cdot5\times10^{-3}$
Schumm & Kahn (1972)	Laboratory	Meandering	1×10^{-3}

TABLE 1. Typical values of β for some natural and laboratory rivers

where

$$A_{11} = M_2(N_1-1),\quad A_{12} = -(M_1+M_2)(M_1+M_2-N_1M_2+N_2M_1),$$
$$A_{21} = -(1+N_2)-(N_1+N_2)M_2+F^{-2}(M_1+M_2-1)(N_1-1),$$
$$A_{22} = -(N_1+N_2)(M_1+M_2)F^{-2},\quad A_3 = F^{-2}(N_1+N_2)(M_1+M_2),\quad A_4 = M_1^2,$$
$$A_{51} = 1,\quad A_{52} = 2(1-M_1)(M_1+M_2)-2M_1(1-F^{-2}),\quad A_{53} = (M_1+M_2)^2,$$
$$A_{61} = -2(1-F^{-2}),\quad A_{62} = (M_1+M_2)^2+(1-F^{-2})^2,\quad A_7 = (1-F^{-2})^2.$$

It is clear that, for this particular solution, non-zero sediment transport and friction ($\beta > 0, C_0 > 0$) are necessary, but not sufficient, conditions for instability.

As instability is observed to occur at coherent finite wavelengths, the observed instability wavelength can be expected to be one at which ϕ_i is positive and a maximum. The stationary points of (15) are obtained by setting $d\phi_i/dk = 0$, yielding

$$\begin{aligned}(B_1K^2+B_2C_0^2)\,k^8+(B_3K^4+B_4C_0^2K^2+B_5C_0^4)\,k^6\\ +(B_6K^6+B_7K^4C_0^2+B_8K^2C_0^4+B_9C_0^6)\,k^4\\ +(B_{10}K^6C_0^2+B_{11}K^4C_0^4)\,k^2+(B_{12}K^8C_0^2+B_{13}K^6C_0^4) = 0,\end{aligned}\qquad (16)$$

where, for example,

$$B_1 = A_3A_{61}-A_7A_{21},\quad B_2 = A_3A_{62}-A_7A_{22}.$$

The above equation has four roots k^2. A general solution is cumbersome, so two complementary special cases based on the parameter $\epsilon = C_0/K$ are considered: $\epsilon \ll 1$ and $\epsilon^{-1} \ll 1$.

For the case $\epsilon \ll 1$ the expansions for two of the four roots are of the form

$$k^2/K^2 = k_W^2(1+\epsilon^2a_1+\ldots). \qquad (17)$$

To lowest order, this implies a relation for the wavelength of the same form as Werner's; thus the roots are termed 'Werner-type'. The expansions for the other two roots are of the form

$$k^2/K^2 = \epsilon k_A^2(1+\epsilon b_1+\ldots), \qquad (18)$$

implying Anderson-type roots. In fact the roots are complementary; the appropriate value of k_A^2 is $(-B_{12}/B_6)^{\frac{1}{2}}$.

For the case $\epsilon^{-1} \ll 1$ two roots are of Hansen type,

$$k^2/C_0^2 = k_H^2(1+\epsilon^{-2}c_1+\ldots), \tag{19}$$

and two have the form

$$k^2/C_0^2 = \epsilon^{-3}k_E^2(1+\epsilon^{-1}d_1+\ldots). \tag{20}$$

The degree of instability occurring at these stationary points can be determined by inserting (17)–(20) into (15) and expanding for ϕ_i. The appropriate expansions are

$$\left.\begin{aligned} \phi_{iW} &= \beta C_0\alpha_W(1+\epsilon^2\alpha_{1W}+\ldots) \quad (21)\\ \phi_{iA} &= \beta C_0\alpha_A(1+\epsilon\alpha_{1A}+\ldots) \quad (22)\end{aligned}\right\} \text{ for } \epsilon \ll 1$$

and

$$\left.\begin{aligned} \phi_{iH} &= \beta C_0\alpha_H(1+\epsilon^{-2}\alpha_{1H}+\ldots) \quad (23)\\ \phi_{iE} &= \beta C_0\epsilon^{-2}\alpha_E(1+\epsilon^{-1}\alpha_{1E}+\ldots) \quad (24)\end{aligned}\right\} \text{ for } \epsilon^{-1} \ll 1.$$

For example, for the Anderson root,

$$\alpha_A = \frac{A_{11}}{A_{51}}, \quad \alpha_{1A} = \left(\frac{A_{21}}{A_{11}} - \frac{A_{61}}{A_{51}}\right)k_A^2 - \frac{A_4}{A_{51}}k_A^{-2}.$$

If the roots (17)–(20) are to represent the characteristic meander or braid wavelengths, they must be real and correspond to positive, maximum instability among all values of k. An exact determination of this requires the assumption of specific constitutive relations; this will be done below for a particular set of data. However, a number of general observations can be made if it is assumed that instability does indeed occur.

In fact the two cases $\epsilon \ll 1$ and $\epsilon^{-1} \ll 1$ can be used to distinguish meandering and braiding regimes. To do this it is first necessary to establish the following result from physical considerations: if any of the wavenumbers given by (17)–(20) corresponds to maximum positive instability in k, then both the first and the second coefficient in the appropriate expansion for ϕ_i are positive. For example, assume that the Anderson-type expansion corresponds to maximum instability. This expansion may be written in the form

$$\phi_{iA} = \beta C_0\alpha_A(1+\epsilon^* m^{-1}\alpha_{1A}+\ldots), \tag{25}$$

where $\epsilon^* = C_0/(\frac{1}{2}F^{-1}k_B)$. Clearly $\alpha_A > 0$. An evaluation of α_{1A} in terms of the coefficients A_{ij} in (15) indicates that α_{1A} does not generally vanish. If α_{1A} is negative, then ϕ_{iA} increases for increasing values of m. Since, for all values of ϵ^*, m can be chosen large enough so that the condition $\epsilon \ll 1$ is satisfied, it follows that for all values of ϵ^* maximum instability occurs when the number of braids is infinite. This contradicts the observed fact that meandering and finitely braided rivers exist, so it must be concluded that $\alpha_{1A} > 0$. A similar argument can be applied to the Werner-type expansions, showing that α_W and α_{1W} are positive.

The Hansen-type expansions can be written in the form

$$\phi_{iH} = \beta C_0\alpha_H(1+m\epsilon^{*-1}\alpha_{1H}+\ldots). \tag{26}$$

Clearly $\alpha_H \neq 0$, and furthermore α_{1H} does not generally vanish. If α_{1H} is negative, then maximum instability occurs for $m = 1$. Thus only meandering occurs, and the tendency to meander increases as ϵ^* becomes large. Note that $\epsilon^* = SB/\pi F d_0$. Thus the tendency to meander increases as the slope increases and as the depth–width ratio decreases. However, this contradicts the observed fact that extremely high slopes and wide, shallow channels are characteristic of braiding. Thus it is concluded that $\alpha_{1H} > 0$. A similar argument can be applied to the coefficients α_E and α_{1E} of the expansions for ϕ_{iE}, showing that both coefficients are positive.

A differentiation between meandering and braided regimes follows from the above. For the exposition, one of the Anderson-type expansions for ϕ_i is assumed to indicate instability in the case $\epsilon \ll 1$ and one of the Hansen-type expansions is assumed to indicate instability in the case $\epsilon^{-1} \ll 1$, although the argument can be repeated with the other expansions. A number ϵ_0 is assumed to exist such that expansions (25) and (26) converge for, respectively, $\epsilon < \epsilon_0$ and $\epsilon^{-1} < \epsilon_0^{-1}$. In (25) decreasing values of m imply increasing instability. The smallest value of m which ensures convergence is $m = \epsilon^*/\epsilon_0$; thus this value of m gives maximum instability among convergent values of m. Among all values of m, then, the value giving maximum instability must satisfy the inequality $m \leqslant \epsilon^*/\epsilon_0$. Applying the same reasoning to (26), the additional condition $m \geqslant \epsilon_0\epsilon^*$ is obtained. Combining these two conditions, the following constraints must be satisfied for maximum instability:

$$\epsilon_0\epsilon^* \leqslant m \leqslant \epsilon^*/\epsilon_0. \tag{27}$$

It was shown previously that m can take only positive integral values. Thus, regardless of the value of ϵ_0, the above condition has the following meaning: for ϵ^* sufficiently small, $m = 1$ and meandering occurs, for ϵ^* sufficiently large braiding occurs, and for $\epsilon^* = O(1)$ either meandering or braiding may occur. An approximate relation for the division between braiding and meandering states is

$$\epsilon^* = \frac{C_0}{\frac{1}{2}F^{-1}k_B} = \frac{SB}{\pi F d_0} = O(1).$$

Clearly then the condition $\epsilon^* \ll 1$ corresponds to extreme meandering, and $\epsilon^* \gg 1$ to extreme braiding. Note that in the case of meandering the condition $\epsilon \ll 1$ is also satisfied since $\epsilon^* = \epsilon$. The condition $\epsilon \gg 1$, however, is not realized even when m is large, as it is apparent from (27) that, as m increases, ϵ^* also increases, so that the scale law $\epsilon = O(1)$ is satisfied.

It follows that meandering instability is characterized by either a Werner- or Anderson-type wavenumber scale. The question as to which scale is in fact observed is quickly resolved on the basis of data: the Werner scale indicates meander wavelengths that are at least an order of magnitude too small. Thus an Anderson-type wavelength relation applies to meandering. None of the wavenumber expansions for the case $\epsilon^{-1} \ll 1$ strictly apply to braiding, since ϵ can never be large enough to assume convergence. However the fact that the magnitude of ϕ_{iE} is much smaller than that of ϕ_{iH} in the region where the expansions are valid indicates that a Hansen-type root (which formally approaches the scale of the Anderson root as $\epsilon \to 1$) provides an appropriate wavelength scale for braiding. Furthermore, (27) indicates that the number of braids is estimated by the scale relation $m \sim \epsilon^*$.

In summary, if $\epsilon^* \ll 1$ meandering occurs at a wavenumber estimated by the relation $k \sim (KC_0)^{\frac{1}{2}}$; if $\epsilon^* \gg 1$ braiding occurs at a wavenumber estimated by $k \sim C_0$ and with a number of braids estimated by $m \sim \epsilon^*$; and if $\epsilon^* = O(1)$ transition between meandering and braiding occurs.

These analytical results concerning meandering and braiding regimes are in agreement with the illuminating qualitative results of Engelund & Skovgaard.

It is interesting to note that, while it is necessary to assume the existence of sediment transport ($\beta > 0$) in this section to obtain instability, the instability wavelength and braid number are independent of β to first order.

8. Establishment of the existence of instability

A verification of the existence of instability requires that the parameters N_1, N_2, M_1 and M_2 appearing in (10) be calculated from specific constitutive relations. For illustrative purposes, this is done herein using the relations of Engelund & Hansen (as reported in Engelund & Skovgaard 1973, pp. 295, 296, 298), which apply to straight sand-bed rivers with dune resistance. In the notation of this paper, the resistance and load relations are, respectively,

$$\theta C_0 F_s^2 = 0{\cdot}06 + 0{\cdot}4(C_0 F_s^2)^2,$$
$$C_0 q^* = 0{\cdot}05(C_0 F_s^2)^{\frac{5}{2}},$$

where θ is defined by the relation $(C_0\theta)^{-\frac{1}{2}} = 6 + 2{\cdot}5 \ln(\theta R/2{\cdot}5)$. After some manipulation it is found that

$$M_2 = (1-\eta)^{-1}, \quad M_1 = 2\xi M_2, \quad N_2 = 1{\cdot}5\eta M_2, \quad N_1 = 3\xi + 2 + 1{\cdot}5\eta M_1,$$

where

$$\eta = \frac{5(\theta C_0)^{\frac{1}{2}}}{[1+5(\theta C_0)^{\frac{1}{2}}](1+\bar{\theta}/\theta)},$$

$$\xi = \frac{1}{[1+5(\theta C_0)^{\frac{1}{2}}](1+\bar{\theta}/\theta)}$$

and

$$\bar{\theta} = -0{\cdot}06(C_0F_s^2)^{-1} + 0{\cdot}4(C_0F_s^2).$$

If C_0 satisfies the inequality $C_0 < 10^{-2}$ (a value rarely exceeded in straight reaches of sand-bed rivers) then it is easy to show that $0 < \eta < \frac{5}{6}$ and $\frac{2}{3} < \xi < 2{\cdot}5$, in which case

$$1 < M_2 < 6, \quad M_1 > M_2, \quad \tfrac{4}{3} < M_1 < 30,$$
$$0 < N_2 < 7{\cdot}5, \quad N_1 > N_2, \quad 4 < N_1 < 47.$$

Expanding (15) about $k = 0$ shows that for small k

$$\phi_i(k) \simeq \beta C_0 \frac{(A_{11}K^4 + A_{12}C_0^2K^2)}{A_4 C_0^2 K^4} k^2.$$

The coefficient A_4 is positive by definition. Furthermore, using the above bounds on M_1, M_2, N_1 and N_2, it is seen that

$$A_{11} = M_2(N_1-1) > 3,$$
$$A_{12} = -(M_1+M_2)[(1-N_1)M_2 + (1+N_2)M_1] = (1+2\xi)(1+\xi)M_2^2 > 0.$$

Thus, under the indicated restrictions, a range of wavenumbers for which ϕ_i is positive always exists and instability characteristic of meandering or braiding always occurs.

9. An interpretation of the criterion for braiding

The physical meaning of the parameter ϵ can be made clear by writing it in a different form:

$$\epsilon(m) = \tau_0 (B/m\pi)/(\rho U^2 d_0)^{\frac{1}{2}} (\rho g d_0^2)^{\frac{1}{2}}. \qquad (28)$$

This is a ratio of work to available energy (with both potential and kinetic energy contributions). Thus the parameter $\epsilon(m)$ can be interpreted to be of the same order as (and a measure of) the ratio of the work that must be done to maintain a mode of oscillation of m braids to the available energy associated with the m-braid mode. If, for some value of m, $\epsilon(m)$ is small enough that the available energy is more than sufficient to overcome resistance, that mode may exist, at least in principle. If $\epsilon(m)$ is large, resistance devours a large portion of m-mode oscillations, and for viable modes the number of braids m must increase, causing ϵ to decrease until the work–energy ratio becomes more favourable. If a number of modes are possible, condition (27) indicates that those with the least excess energy are most likely to occur.

The above has interesting implications for channel form. It has often been observed that the banks of braided rivers are generally straighter than those of meandering rivers (e.g. see the remark concerning Lane's hypothesis on p. 1298 of Vanoni *et al.* 1972). It is reasonable to assume that bank erosion due to meandering and braiding is caused the by excess energy of transverse oscillation. Thus, for any mode of m braids, if ϵ can be made small, significant local bank erosion may occur. However, in the case of braided rivers, ϵ simply cannot be small. If, for some m, $\epsilon = C_0/(\frac{1}{2}mF^{-1}k_B)$ is made sufficiently small, m will be reduced until modes with smaller excess energy are obtained; that is, instead of the banks being eroded, the number of braids is reduced. The only case in which this cannot happen is that of $m = 1$, the case of meandering. Since the number of braids cannot be reduced in this case, excess energy is available for bank erosion. Bank erosion then will continue, increasing sinuosity and thus the comparative effect of friction, until again the excess potential energy is minimized. Thus greater eventual sinuosity is expected for smaller initial ϵ^* ($= \epsilon$ when $m = 1$).

The fact that meandering in laboratory streams is generally not very sinuous can be explained as follows. Since loose sand is generally used for bed and bank material, the banks are extremely erodible, and the stream widens so much before fluvial instability occurs that ϵ^* is increased considerably. The above theory predicts that significant sinuosity can be attained in the laboratory by constructing a relatively narrow channel (in so far as the condition $d_0/B \ll 1$ is satisfied) with loose sand for the bed material, and for the banks a partly cohesive material that, while erodible, is less erodible than that on the bed.

10. Remarks on the bedload-independent solutions

The development of the previous two sections hinged on the assumption that instability was dependent on sediment transport in a dynamic way. However, many authors have asserted that meandering is a property of the flow, the role of sediment transport being to kinematically and passively make this instability visible on the bed. From this point of view the role of sediment in meandering is similar to the role of dye placed in turbulent water; the dye does not cause the turbulence, but rather merely serves to make it manifest.

One way to test this assumption is to see whether flow instability on the scale of meandering exists in non-erodible channels. This can be done by examining the roots of the 'outer' characteristic equation (13), in which C_0 has been equated to the grain resistance C_G in recognition of the fact that no sediment transport implies no bedforms. For an examination of meandering, m is set equal to 1 and the approximation $\epsilon^* \ll 1$ (associated with meandering) is applied. Expansions of the form $\phi = a_1 + C_0^2 a_2 + \ldots + iC_0(b_1 + C_0^2 b_2 + \ldots)$ can be obtained for the three roots of (13); to the lowest order that allows for a decision on stability, they are

$$\phi_1 = k - iC_G\left(\frac{K^2\bar{M}_1 + F^{-2}k^2}{K^2 + F^{-2}k^2}\right),$$

$$\phi_2 = k - (K^2 + F^{-2}k^2)^{\frac{1}{2}} - iC_G\left[\frac{\bar{M}_1 F^{-2}k^2 + K^2 + \bar{M}_2 k(K^2 + F^{-2}k^2)^{\frac{1}{2}}}{2(F^{-2}k^2 + K^2)}\right],$$

$$\phi_3 = k + (K^2 + F^{-2}k^2)^{\frac{1}{2}} - iC_G\left[\frac{\bar{M}_1 F^{-2}k^2 - \bar{M}_2 k(K^2 + F^{-2}k^2)^{\frac{1}{2}}}{2(F^{-2}k^2 + K^2)}\right],$$

where

$$\bar{M}_1 = 2\left(1 + \frac{1}{2}\frac{U}{C_G}\frac{\partial C_G}{\partial U}\right)\left(1 - \frac{S}{C_G}\frac{\partial C_G}{\partial S}\right)^{-1},$$

$$\bar{M}_2 = \left(1 - \frac{S}{C_G}\frac{\partial C_G}{\partial S}\right)^{-1}.$$

If C_G were completely constant, $\bar{M}_1$ would equal 2 and $\bar{M}_2$ would equal 1; actual values roughly approximate these. Clearly the first and second roots indicate stability; the third root indicates instability for only

$$F > \frac{\bar{M}_1 k^2 + \frac{1}{4}k_B^2}{\bar{M}_2 k^2 + \frac{1}{4}k_B^2} \geqslant \frac{\bar{M}_1}{\bar{M}_2} \sim 2.$$

This implies a meander-like analogy to the Vedernikov or roll-wave instability (see Chow 1959; Henderson 1966, p. 342), which has nothing to do with fluvial meandering. Allowing C_G to vary according to a logarithmic law does not change the qualitative behaviour, but simply alters somewhat the Froude number at which Vedernikov instability occurs.

Thus the following result has been established: if ϵ^* is sufficiently small, meandering is not a property of the flow, and non-zero sediment transport is a necessary condition for the formation of instability leading to meandering. Furthermore, it is then apparent that, in the presence of sediment transport, small ϵ^* is a sufficient condition for meandering if any fluvial instability exists at all. [See (27).] A similar result can be established for the case $\epsilon \gg 1$, again indicating a critical Froude number near 2 below which instability does not occur.

11. Comparison with data

The parameter ϵ^* can be written in the form

$$\epsilon^* = \frac{1}{\pi}\frac{S}{F}\frac{B}{d_0}.$$

The theory indicates that, in rivers such that sediment transport exists and $d_0/B \ll 1$ at formative discharges (both conditions are almost universally satisfied in natural rivers), a tendency towards either meandering or braiding exists. Furthermore, meandering occurs for $S/F \ll d_0/B$, braiding occurs for $S/F \gg d_0/B$ and transition between the two occurs for $S/F \sim d_0/B$.

These relationships corroborate the observation that, while meandering streams usually have gentle slopes and rather narrow channels, braided streams generally have steep slopes and wide channels. Thus the same river may often be braided in its mountain reaches but meandering in its more gently sloping lower reaches, as Chien (1961) has observed on the Yellow River. Schumm (1963) has observed that the depth–width ratio tends to decrease with decreasing bank cohesivity; this combined with the above stability criteria helps to explain why some rivers meander and others braid at the same slope.

The theory does not indicate any conditions under which a stream which transports sediment remains straight. Chang, Simons & Woolhiser (1971) and Vincent (1967) have demonstrated experimentally that for sufficiently large values of d_0/B neither meandering nor braiding develops. The condition for the maintenance of a straight channel appears to be $d_0/B > 10^{-1}$. Such large values of d_0/B can be attained in the laboratory and in artificial canals but are rarely attained in natural rivers. Anderson *et al.* (1975) have shown that in narrow channels the augmented effect of bank friction tends to damp instability.

Thus the results of this and other analyses can be combined to delineate an order-of-magnitude meander/braid/straight regime diagram, which is tested in figure 4. The data cover 75 laboratory flume experiments, 22 irrigation canals and reaches of 53 natural rivers. The natural rivers are referenced in table 2. Where possible, field data estimated for bank-full flow were used with the implicit assumption that the morphologically formative discharge ranges are near (probably above) bank-full discharge. The relative lack of such data required the inclusion of data from rivers based on mean or other flow conditions. The discrepancies thus introduced do not appear to be critical; for example, point 3 of figure 4 represents a reach of the Mississippi River at mean discharge, whereas point 4 represents the same reach at bank-full flow. Another source of inaccuracy is the fact that some data had to be estimated from information in the literature that was occasionally rather crude.

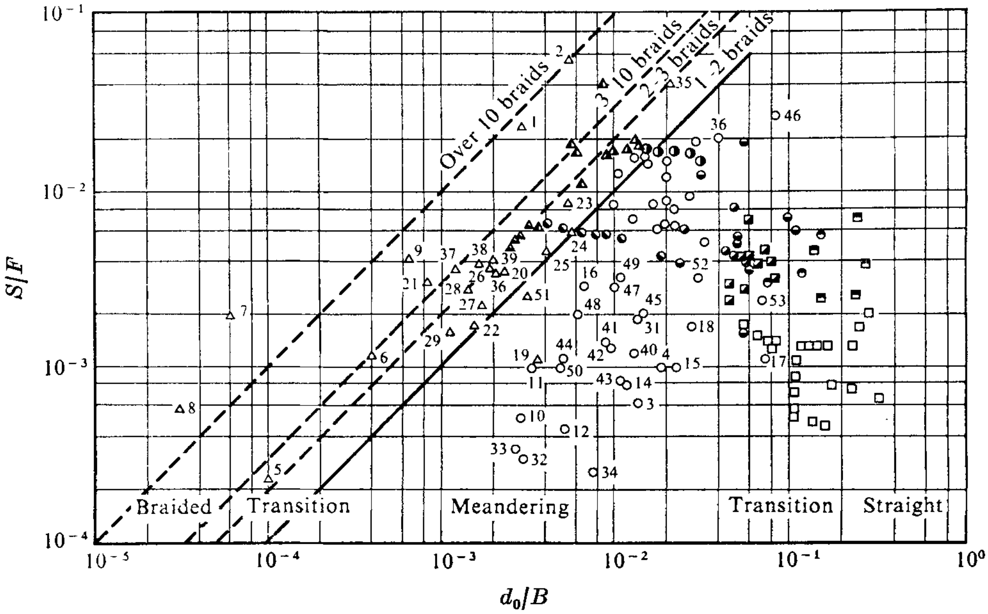

FIGURE 4. Meander/braid/straight regime diagram, tested with data. The triangles mark stream reaches that were actually braided, circles mark meandering reaches and squares mark straight streams. Laboratory experiments: ◑, ▲, St Anthony Falls (run M-1-1); ◭, St Anthony Falls braided data; ◪, ◔, Wolman & Brush (1961); ▲, ◕, Schumm & Kahn (1972); ◒, ▬, Ashida & Narai (1969); ○, Ackers & Charlton (1970); ◓, Quraishy (1973). Field observations: □, Simons (1957), irrigation canals; △, ○, rivers (numbers refer to table 2).

Equation (18) can be written in the following form, providing a predictive relation for meander length:

$$\frac{\lambda}{(Bd_0)^{\frac{1}{2}}} = 2\psi\pi^{\frac{1}{2}}C_0^{-\frac{1}{2}}F^{\frac{1}{2}}, \tag{29}$$

where

$$\psi = \left[\frac{(1+N_2)(1+M_2)+M_2(1-N_1)+F^{-2}(1-N_1)(M_1-M_2-1)}{M_2M_1^2(N_1-1)}\right]^{\frac{1}{4}}.$$

Note that this is the form of the Anderson equation with an evaluation of the adjustable constant that Anderson obtained as roughly equal to 72. Hereafter the above will be referred to as the modified Anderson equation. The instability associated with the predicted wavelength is measured by the imaginary part of the complex celerity $\phi_i = \beta C_0 M_2(N_1-1)$, which must be positive.

The meandering characteristics λ and ϕ_i, then, are dependent not only on the relatively easily measured parameters B, d_0, F, C_0 and β, but also on the parameters

$$\frac{1}{2}\frac{U}{C_0}\frac{\partial C_0}{\partial U}, \quad \frac{S}{C_0}\frac{\partial C_0}{\partial S}, \quad \frac{U}{q_0}\frac{\partial q_0}{\partial U}, \quad \frac{S}{q_0}\frac{\partial q_0}{\partial S}.$$

In general, none of the latter parameters is negligible.

The results concerning meandering may be verified or invalidated by experiments that are relatively simple in principle. It has been pointed out previously that meandering is observed to occur in flumes corresponding to the model

Point	River	Discharge conditions	Location	Source
1	White River	Instantaneous	Mt Ranier, USA	Fahnestock (1963)
2	White River	Instantaneous	Mt Ranier, USA	Fahnestock (1963)
3	Mississippi River	Mean	Vicksburg, USA	Winkley (1973)
4	Mississippi River	Bank full	Vicksburg, USA	Winkley (1973)
5	Yellow River	Near bank full	Mengtsing, China	Chien (1961)
6	Glomma River	Bank full	Koppangsoyene, Norway	Nordseth (1973)
7	Matanuska River	Normal summer high	Anchorage, USA	Fahnestock & Bradley (1973)
8	Knik River	Normal summer high	Anchorage, USA	Fahnestock & Bradley (1973)
9	Knik River	Catastrophic flood	Anchorage, USA	Fahnestock & Bradley (1973)
10	Missouri	High	Pierre, S. Dak., USA	Einstein & Barbarossa (1952)
11	Missouri	Low	Pierre, S. Dak., USA	Einstein & Barbarossa (1952)
12	Missouri	High	Ft Randall, USA	Einstein & Barbarossa (1952)
13	Missouri	Low	Ft Randall, USA	Einstein & Barbarossa (1952)
14	Missouri	—	Omaha, Nebr., USA	Einstein & Barbarossa (1952)
15	Elkhorn	High	Waterloo, USA	Einstein & Barbarossa (1952)
16	Elkhorn	Low	Waterloo, USA	Einstein & Barbarossa (1952)
17	Big Sioux	High	Akron, USA	Einstein & Barbarossa (1952)
18	Big Sioux	Low	Akron, USA	Einstein & Barbarossa (1952)
19	Platte	High	Ashland, USA	Einstein & Barbarossa (1952)
20	Platte	Low	Ashland, USA	Einstein & Barbarossa (1952)
21	Niobrara	—	Butte, USA	Einstein & Barbarossa (1952)
22	Salinas	High	San Lucas, USA	Einstein & Barbarossa (1952)
23	Salinas	Low	San Lucas, USA	Einstein & Barbarossa (1952)
24	Salinas	High	Paso Roblis, USA	Einstein & Barbarossa (1952)
25	Salinas	Low	Paso Roblis, USA	Einstein & Barbarossa (1952)
26	Middle Loup	Instantaneous	Dunning, USA	Hubbell & Matejka (1959)
27	Middle Loup	Instantaneous	Dunning, USA	Hubbell & Matejka (1959)
28	Middle Loup	Instantaneous	Dunning, USA	Hubbell & Matejka (1959)
29	Middle Loup	Instantaneous	Dunning, USA	Hubbell & Matejka (1959)
30	Middle Loup	Instantaneous	Dunning, USA	Hubbell & Matejka (1959)
31	Ganga	Bank full	Kankhal, India	Gupta *et al.* (1969)
32	Great Gandak	Bank full	Chitauni, India	Gupta *et al.* (1969)
33	Sarda	Bank full	Sarda Sagar, India	Gupta *et al.* (1969)
34	Ganga	Bank full	Raighat Narora, India	Gupta *et al.* (1969)
35	Beaver Creek	Mean	Daniel, Wyo., USA	Leopold & Wolman (1957)
36	Watts Branch	Mean	Rockville, Md., USA	Leopold & Wolman (1957)
37	Platte	Mean	Grand Island, Nebr., USA	Leopold & Wolman (1957)
38	Platte	Mean	Odessa, Nebr., USA	Leopold & Wolman (1957)
39	Platte	Mean	Overton, Nebr., USA	Leopold & Wolman (1957)
40	Missouri	Mean	St Joseph, Mo., USA	Leopold & Wolman (1957)
41	Missouri	Mean	Hermann, Mo., USA	Leopold & Wolman (1957)
42	Missouri	Mean	Pierre, S. Dak., USA	Leopold & Wolman (1957)
43	Missouri	Mean	Kansas City, Mo., USA	Leopold & Wolman (1957)
44	Missouri	Mean	Bismarck, N. Dak., USA	Leopold & Wolman (1957)
45	Tennessee	Mean	Knoxville, Tenn., USA	Leopold & Wolman (1957)
46	Buttahatchee	Mean	Caledonia, Miss., USA	Leopold & Wolman (1957)
47	Kansas	Mean	Bonner Springs, Kan., USA	Leopold & Wolman (1957)
48	Kansas	Mean	Lecompton, Kan., USA	Leopold & Wolman (1957)
49	Kansas	Mean	Ogden, Kan., USA	Leopold & Wolman (1957)
50	Maumee	Mean	Defiance, Ohio, USA	Leopold & Wolman (1957)
51	Maumee	Mean	Waterville, Ohio, USA	Leopold & Wolman (1957)
52	Smoky Hill	Mean	Lindsborg, Kan., USA	Leopold & Wolman (1957)
53	Smoky Hill	Mean	Enterprise, Kan., USA	Leopold & Wolman (1957)

TABLE 2. Key to natural rivers in figure 4

theoretical channel postulated herein and that, furthermore, essentially steady-state flow with developed grain and bedform resistance occurs long before meandering begins to occur. In accordance with the concept of stability, all the appropriate parameters except the meander length itself must be obtained by measuring this initial non-meandering flow, the derivatives being obtained by comparing steady-state initial flows of slightly different discharge and slope.

Unfortunately, such experiments have not yet been conducted. (Perhaps the experiments of Callander (1969) constitute the closest attempt.) However, data are available for the flow conditions which exist after meandering has developed fully. Such data are inappropriate for studying the origin of meandering, as they contain the extra resistance of the meanders themselves, which cannot exist prior to meandering. Thus such data are used herein with the aid of the unverifiable and possibly inadequate assumption that the large-scale bedform resistance is negligible. An example of such data is contained in the work of Chang *et al.* (1971), conducted in a flume corresponding to the model theoretical channel. Their experiments were conducted with three bed materials: sand, expanded clay and plastic pellets.

A comparison with the Chang *et al.* data, keeping in mind the difficulty cited previously, was attempted. Apparently because the clay and sand experiments were conducted in the transition regime between hydraulically smooth and hydraulically rough flow, a satisfactory division of the bed resistance into grain and bedform components C_G and C_B could not be accomplished; thus only the plastic experiments, approaching fully rough flow, were used.

Almost all the plastic experiments were conducted in a 3 ft wide flume with plastic pellets having a uniform characteristic diameter estimated to be 3·2 mm. C_G was obtained from a solution of a logarithmic resistance equation for rough flow. In the same way the appropriate derivatives of C_G can be obtained:

$$\frac{1}{2}\frac{U}{C_G}\frac{\partial C_G}{\partial U} = -\frac{S}{C_G}\frac{\partial C_G}{\partial S},$$

$$\frac{S}{C_G}\frac{\partial C_G}{\partial S} = \frac{1}{(1+0{\cdot}202C_G^{-\frac{1}{2}})}.$$

In figure 5, the bedform resistance C_B has been plotted against the flow velocity U, the curve beginning from a velocity estimated from Shield's criterion as the velocity at the inception of sediment motion. Characteristic rising, plateau and falling regimes of bedform resistance are apparent (Raudkivi 1963). As the data were too few to enable both derivatives of C_B to be obtained accurately, it was decided to obtain them from at least the form of a known bedform resistance relation. The relation of Shen (1962), it was found, could be modified to represent the data in a rough way for the regime of falling resistance, as is illustrated in figure 6; thus the three points not in this regime were discarded and the derivatives of C_B for the remaining points obtained from

$$\frac{1}{2}\frac{U}{C_B}\frac{\partial C_B}{\partial U} = -0{\cdot}096C_B^{-\frac{1}{2}}\left(1+\frac{1}{2}\frac{U}{C_G}\frac{\partial C_G}{\partial U}\right),$$

$$\frac{S}{C_B}\frac{\partial C_B}{\partial S} = -0{\cdot}096\,C_B^{-\frac{1}{2}}\frac{S}{C_G}\frac{\partial C_G}{\partial S}.$$

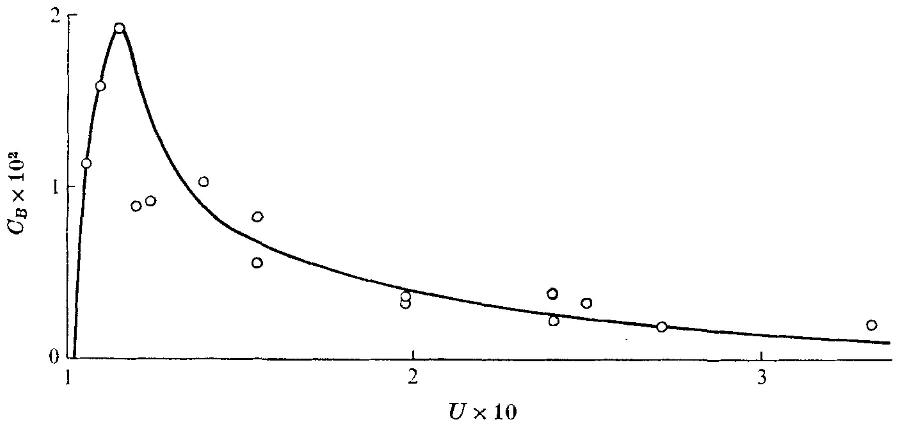

FIGURE 5. Bedform resistance coefficient C_B plotted against average flow velocity U (in m/s), for the Chang *et al.* (1971) experiments with plastic bed materials in a 3 ft wide flume.

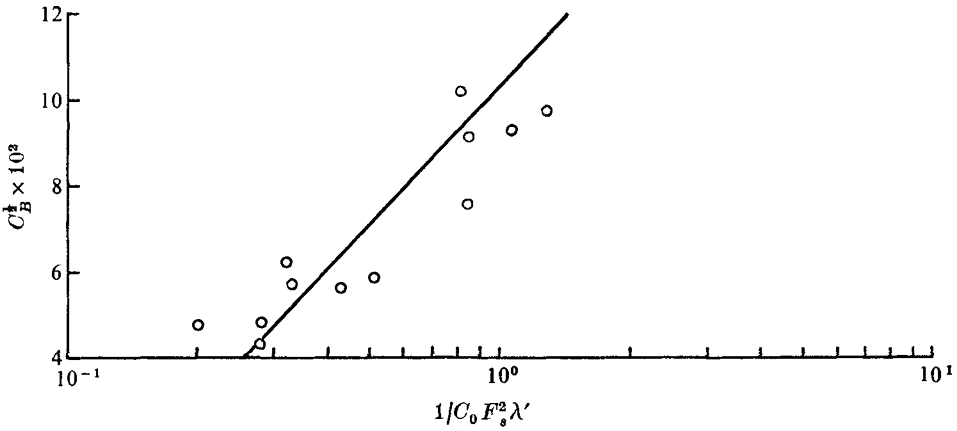

FIGURE 6. A crude fit of some of the Chang *et al.* (1971) plastic data with a Shen-type bedform resistance relation:

$$C_B = [0{\cdot}10 - 0{\cdot}11 \ln (C_0 F_s^2 \lambda')]^2,$$

where $\lambda' = (v_s d_s/\nu)^{\frac{1}{2}}$ and v_s is grain fall velocity.

Bedload parameters were found in a similar way assuming that the Einstein–Brown bedload relation (Brown 1950) holds.

In this manner, values of $\lambda/(Bd_0)^{\frac{1}{2}}$ predicted by (29), as well as experimental values, were calculated; these are compared in figure 7. For reference purposes, observed values of λ/d_0 are plotted against values predicted by Hansen's theoretical relation $\lambda/d_0 = 7C_0^{-1}$ in figure 8. For this set of data the modified Anderson equation appears to be the more accurate of the two. It must, however, be realized that the nature and sparseness of the data preclude a thorough comparison of the two relations.

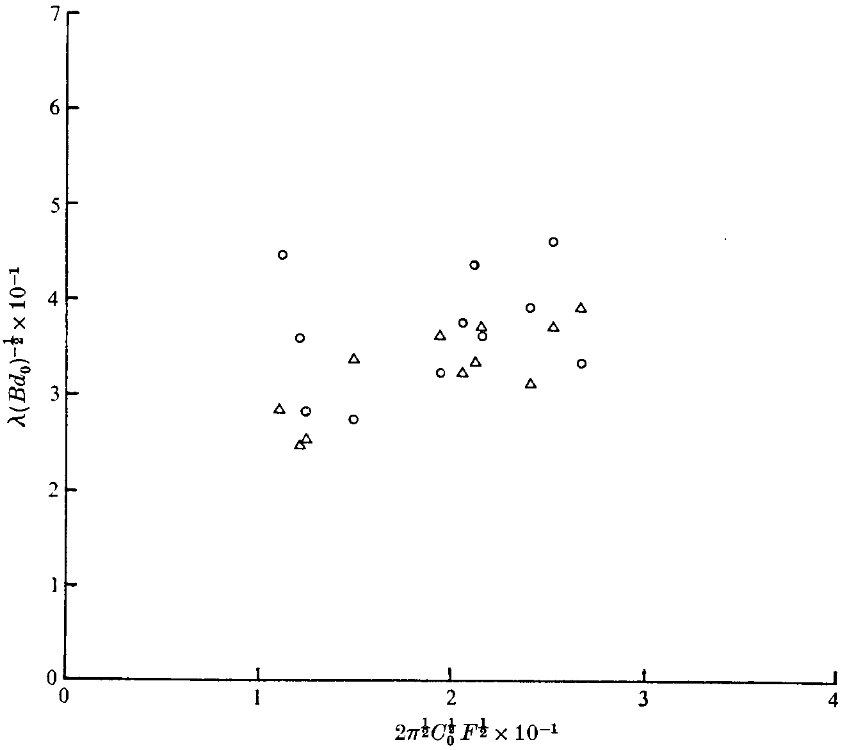

FIGURE 7. According to the modified Anderson relation, the dimensionless meander length $\lambda/(Bd_0)^{\frac{1}{2}}$ is scaled by the parameter $2\pi^{\frac{1}{2}}C_0^{-\frac{1}{2}}F^{\frac{1}{2}}$ and can be calculated to first order from

$$\lambda/(Bd_0)^{\frac{1}{2}} = 2\pi^{\frac{1}{2}}\psi C_0^{-\frac{1}{2}}F^{\frac{1}{2}},$$

where ψ is a complicated order-one coefficient. Observed (circles) and calculated (triangles) values of $\lambda/(Bd_0)^{\frac{1}{2}}$ are plotted against $2\pi^{\frac{1}{2}}C_0^{-\frac{1}{2}}F^{\frac{1}{2}}$. Perfect agreement implies coincidence of the circles and triangles.

12. Conclusion and discussion

Meandering and braiding are treated as different degrees of the same instability phenomenon. Sediment transport and friction are indicated to be essential factors for the occurrence of instability, whereas helicity is not essential. An Anderson-type relation for meander wavelength, a criterion for meandering and braided regimes, and an estimate of the number of braids are obtained; all these results are essentially independent of the magnitude of sediment transport.

The unified treatment of meandering and braiding of this and other studies deserves justification in terms of geomorphology. Both phenomena clearly grow out of bar formation. It has traditionally been held that braiding is caused by sediment loads so high that the river cannot carry the total amount, resulting in deposition on the bed as internal bars and general channel aggradation. On the other hand, the mechanism causing meandering is typically identified as secondary flow associated with channel curvature. If the causes of meandering and braiding were so different a unified approach would be impossible.

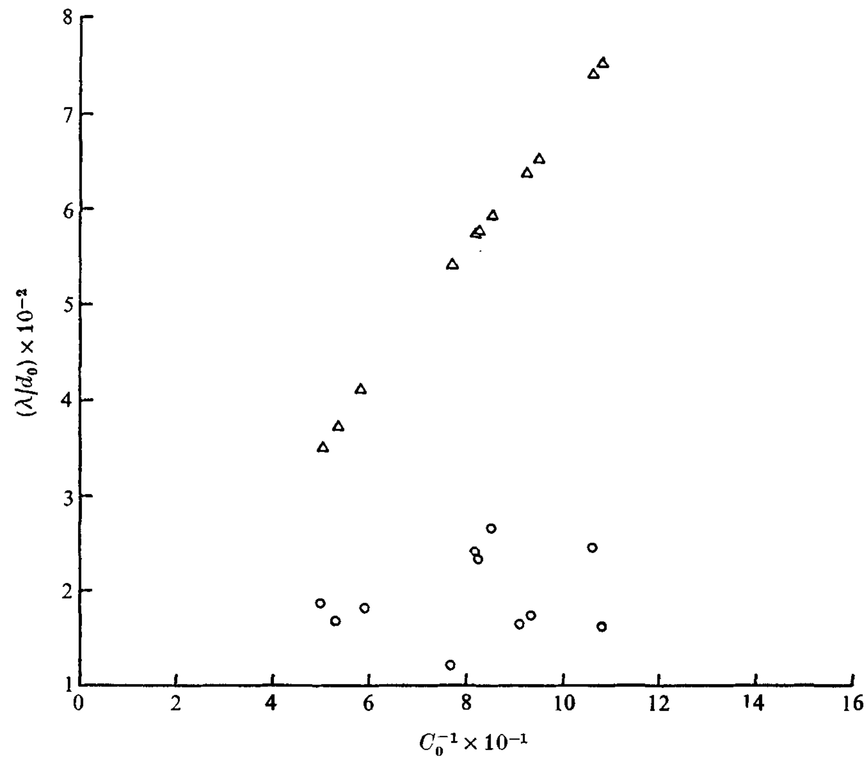

FIGURE 8. According to the Hansen relation, the dimensionless meander length λ/d_0 is scaled by the parameter C_0^{-1} and can be calculated from $\lambda/d_0 = 7C_0^{-1}$. Observed (circles) and calculated (triangles) values of λ/d_0 are plotted against C_0^{-1}. Perfect agreement implies coincidence of the circles and triangles.

In fact both these theories are demonstrably incorrect. The first theory implies that braided channels can never be in an equilibrium, or graded state, whereby the load supplied from upstream of a point is balanced by the load transported downstream. Presumably, then, aggradation occurs until a higher, equilibrium slope is obtained, at which point braiding must stop. However, slope increases are in fact observed to exacerbate braiding rather than damp it. Furthermore, many braided rivers do not aggrade: Leopold & Wolman (1957) have observed that "...braided patterns may be as close to quasi-equilibrium as rivers possessing meandering or other patterns". As regards meandering, it has been shown herein that the channel curvature needed to induce secondary flow is a result rather than a cause of initial meandering tendencies in straight channels, a fact that has been experimentally verified (see §2).

The analysis of this paper indicates that most streams have a tendency to form bars even though they are in a graded state. If the slope and the width–depth ratio at formative discharges are sufficiently low, meandering is favoured. If the slope and the width–depth are sufficiently high, braiding is favoured. The fundamental question of how slope, width and depth are determined is not addressed herein; it suffices to observe that aggradation, by increasing the slope and forcing the channel out of its banks, can lead to a transition from a meandering to a braided state, or can increase the tendency for braiding.

The result concerning sediment transport is also of some interest. The thesis that meandering is an inherent property of the flow, and that sediment transport is necessary only in a kinematic way to impose this flow pattern on the bed, must be discarded in so far as the present theory applies. Rather, it is indicated that the existence of sediment transport is a dynamically necessary condition for the formation of instability leading to meandering either in the flow or on the bed.

This conclusion must be reconciled with the fact that meandering in fluid streams occurs in circumstances in which sediment transport is not present; namely, in oceanic currents such as the Gulf Stream, streams of meltwater on ice, and Gorycki's streams a few millimetres wide on plastic plates. Common to all meandering streams are potential (inertial and gravitational) and friction effects; it is proposed here that an additional 'third effect' is required for meandering. This third effect is identified as follows: for alluvial streams, sediment transport; for oceanic currents, the Coriolis acceleration; for glacial meltwater streams, heat differences, and for Gorycki's streams, surface tension. In the first three cases, this identification is supported by, respectively, this paper, Stommel (1965) and Parker (1975). Gorycki's reply (1973*b*) to discussion on his paper also contains a note on the role of surface tension.

Perhaps the major inadequacy of the present theory is that the channel width at a formative discharge must be known before any of the relations can be evaluated. This and the fact that constant discharge is assumed make application to natural rivers difficult. The theoretical study of Hirano (1973) on bank erosion and the experimental work of Ackers & Charlton (1970) on the dominant discharge of meandering rivers provide examples of possible approaches to these problems.

The author deeply thanks Dr Alvin Anderson for suggestions and guidance. Also, suggestions of several anonymous referees have proved valuable and have been incorporated into the text. This research was supported by the National Science Foundation and the St Anthony Falls Hydraulic Laboratory.

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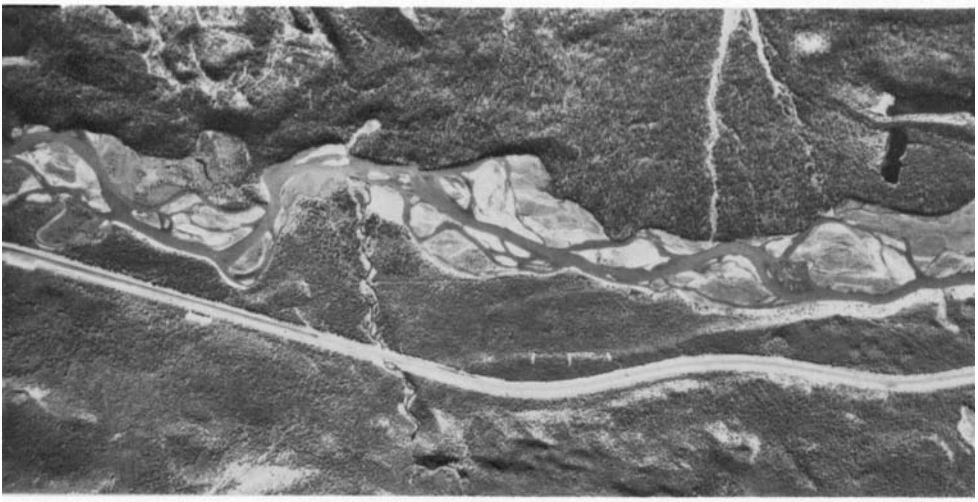
(*a*)



(*b*)

FIGURE 1. (*a*) A highly meandering river reach: the Pembina River near Jarvie, Alberta, Canada. (*b*) A moderately braided river reach: the North Saskatchewan River near Banff National Park, Alberta, Canada.

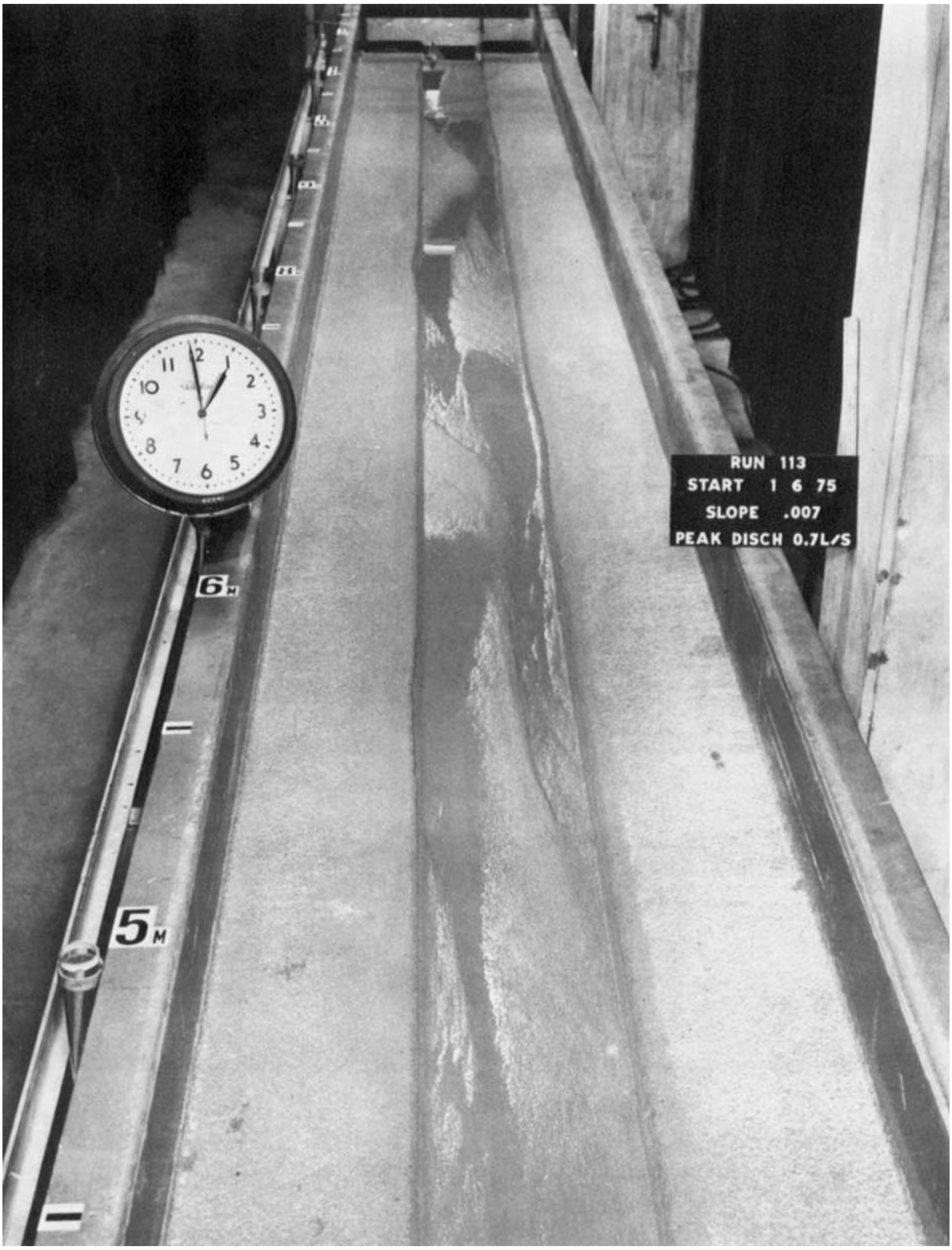
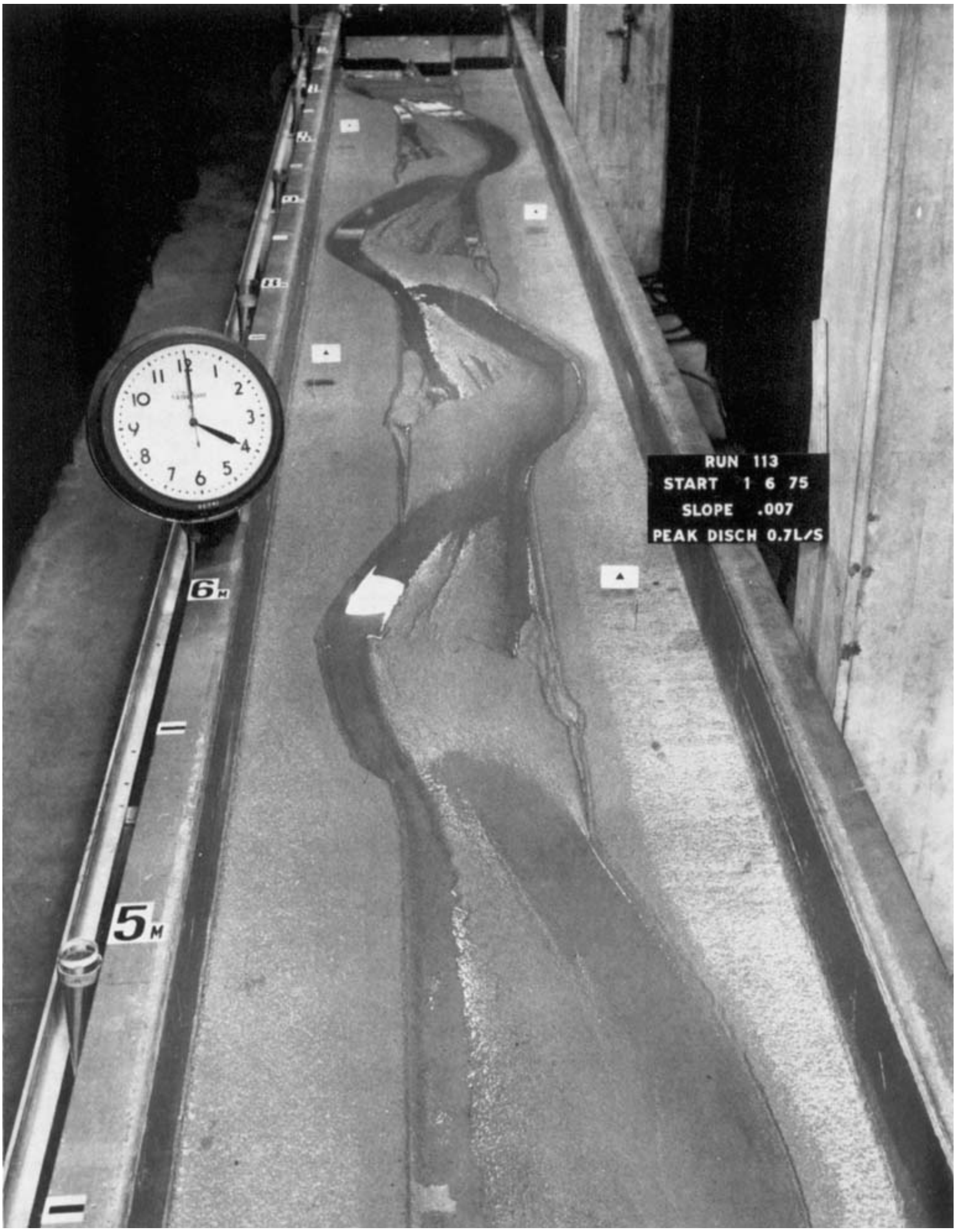


FIGURE 2(*a*). For legend see facing page.



(*b*)

FIGURE 2. Photographs of progressive meander formation in an initially straight channel. Flow is from bottom to top; discharge has been momentarily lowered to render bed patterns visible. (*a*) One hour after start. (*b*) Four hours after start.